

Quiz 3

1. Describe the given set in spherical coordinates. Be sure to include clear bounds for each spherical coordinate AND either sketch or give a detailed description of the set using words. (1)

$$x^2 + y^2 + z^2 \leq 1, x = y, x \geq 0, y \geq 0 \quad (2)$$

$$x^2 + y^2 + z^2 \leq 1 \Rightarrow \rho^2 \leq 1 \Rightarrow \rho \leq 1.$$

$$x = y \Rightarrow \rho \cos \theta \sin \phi = \rho \sin \theta \sin \phi$$

$$\Rightarrow \cos \theta = \sin \theta \Rightarrow \theta = \frac{\pi}{4} \text{ since } x \geq 0, y \geq 0.$$

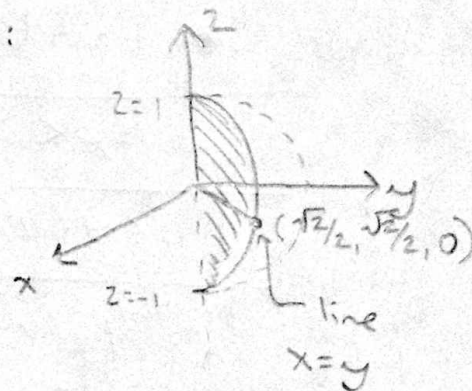
So, the bounds are:

$$0 \leq \rho \leq 1$$

$$\theta = \pi/4$$

$$0 \leq \phi \leq \pi$$

(2) This is a half disk in the octants where $x \geq 0, y \geq 0$ centered at $(0,0,0)$ and passing through $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$ and with radius 1



2. What is the center of the circle with the following parametrization?

$$r(t) = (-2 + \cos t)i + 2j + (3 - \sin t)k$$

$$(-2, 2, 3)$$

Extra Practice

Describe the given set in spherical coordinates:

$$x^2 + y^2 = 3z^2$$

Substituting the spherical coordinates we get:

$$\rho^2 \cos^2 \theta \sin^2 \phi + \rho^2 \sin^2 \theta \sin^2 \phi = 3\rho^2 \cos^2 \phi.$$

$\rho = 0$ is one solution, otherwise we have:

$$\sin^2 \phi (\cos^2 \theta + \sin^2 \theta) = 3 \cos^2 \phi \Rightarrow$$

$$\sin^2 \phi = 3 \cos^2 \phi \Rightarrow$$

$$\frac{\sin^2 \phi}{\cos^2 \phi} = 3 \Rightarrow \tan \phi = \sqrt{3} \quad \text{or} \quad \tan \phi = -\sqrt{3}$$

$$\Rightarrow \phi = \frac{\pi}{3} \quad \text{or} \quad \phi = \frac{2\pi}{3}$$

Hence the bands are:

$0 \leq \rho < \infty$
$0 \leq \theta < 2\pi$
$\phi = \frac{\pi}{3}$ or $\phi = \frac{2\pi}{3}$

So, the set is the surface obtained by rotating a line that makes an angle of $\frac{\pi}{3}$ about the z -axis. It is a double cone.

Sketch

